**Quadratic inequalities**

**A LEVEL LINKS**

**Scheme of work:** 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

* First replace the inequality sign by = and solve the quadratic equation.
* Sketch the graph of the quadratic function.
* Use the graph to find the values which satisfy the quadratic inequality.

Examples

**Example 1** Find the set of values of *x* which satisfy *x*2 + 5*x* + 6 > 0

|  |  |
| --- | --- |
| *x*2 + 5*x* + 6 = 0  (*x* + 3)(*x* + 2) = 0  *x* = −3 or *x* = −2    *x* < −3 or *x* > −2 | **1** Solve the quadratic equation by factorising.  **2** Sketch the graph of  *y* = (*x* + 3)(*x* + 2)  **3** Identify on the graph where  *x*2 + 5*x* + 6 > 0, i.e. where *y* > 0  **4** Write down the values which satisfy the inequality *x*2 + 5*x* + 6 > 0 |

**Example 2** Find the set of values of *x* which satisfy *x*2 − 5*x* ≤ 0

|  |  |
| --- | --- |
| *x*2 − 5*x* = 0  *x*(*x* − 5) = 0  *x* = 0 or *x* = 5    0 ≤ *x* ≤ 5 | **1** Solve the quadratic equation by factorising.  **2** Sketch the graph of *y* = *x*(*x* − 5)  **3** Identify on the graph where  *x*2 − 5*x* ≤ 0, i.e. where *y* ≤ 0  **4** Write down the values which satisfy the inequality *x*2 − 5*x* ≤ 0 |

**Example 3** Find the set of values of *x* which satisfy −*x*2 − 3*x* + 10 ≥ 0

|  |  |
| --- | --- |
| −*x*2 − 3*x* + 10 = 0  (−*x* + 2)(*x* + 5) = 0  *x* = 2 or *x* = −5    −5 ≤ *x* ≤ 2 | **1** Solve the quadratic equation by factorising.  **2** Sketch the graph of *y* = (−*x* + 2)(*x* + 5) = 0  **3** Identify on the graph where −*x*2 − 3*x* + 10 ≥ 0, i.e. where *y* ≥ 0  **3** Write down the values which satisfy the inequality −*x*2 − 3*x* + 10 ≥ 0 |

Practice

**1** Find the set of values of *x* for which (*x* + 7)(*x* – 4) ≤ 0

**2** Find the set of values of *x* for which *x*2 – 4*x* – 12 ≥ 0

**3** Find the set of values of *x* for which 2*x*2 –7*x* + 3 < 0

**4** Find the set of values of *x* for which 4*x*2 + 4*x* – 3 > 0

**5** Find the set of values of *x* for which 12 + *x* – *x*2 ≥ 0

Extend

Find the set of values which satisfy the following inequalities.

**6** *x*2 + *x* ≤ 6

**7** *x*(2*x* – 9) < –10

**8** 6*x*2 ≥ 15 + *x*

Answers

**1** –7≤ *x* ≤ 4

**2** *x* ≤ –2 or *x* ≥ 6

**3 **

**4** *x* <  or *x* > 

**5** –3 ≤ *x* ≤ 4

**6** –3 ≤ *x* ≤ 2

**7** 2 < *x* < 2

**8**  or 